Laplace Equation

Laplace's equation is a second order partial differential equation[1] of potential theory. It is most often written as

$$\nabla^2\varphi = 0\,,$$

where ∇^2 is the Laplace operator[2]. Solutions of Laplace's equation are often called harmonic functions.

Laplace's Equation is a special case of Poisson's equation[3]; the latter tends to apply to domains that include sources whereas Laplace's equation is generally applicable in regions where there is no source. Laplace's equation has a wide range of applications: steady-state heat conduction[4], electrostatics[5], groundwater flow[6], gravitation[7] and ideal fluid flow[8].

Although the Laplace equation can be theoretically of any number of dimensions, in applied mathematics, it is of interest mainly in three dimensions or two dimensions. In two dimensions the Laplace equation has the we have:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0,$$

and in three dimension,

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0\,.$$

Laplace's equation on its own has many solutions. For example $\varphi = xy$ and $\varphi = x^2 - y^2$ are solutions of the two-dimensional Laplace equation and $\varphi = xyz$ and $\varphi = 2x^2 - y^2 - z^2$ are solutions of the three-dimensional Laplace equation.

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2 Laplace Operator or Laplacian
3 Poisson's Equation
4 Heat Conduction Model as a Partial Differential Equation
5 Electrostatics Model as a Partial Differential Equation
6 Groundwater Flow Model as a Partial Differential Equation
7 Gravitation Model as a Partial Differential Equation
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